Coupling CFD / experiments : implementation of a Kalman filter based approach

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Research activity performed using OpenFOAM

1. Uncertainty Quantification (UQ) for turbulent flows: L. Margheri, M. V. Salvetti, P. Sagaut
2. Immersed Boundary Method (IBM): H. Riahi, E. Goncalves, E. Constant, P. Meliga, J. Favier, E. Serre
3. **Estimation Theory (ET): F. Gava, A. Poux, J. Borée, G. Lehnasch, GANTHA**

Turbulent flows

Turbulence is present in most engineering applications.

A complete and univoque definition of turbulence is still missing.

Turbulence characteristics

A turbulent flow is:

- driven by non linear dynamics
- unstationary
- extremely sensitive to boundary and initial conditions

Comparison experiments / CFD difficult:

1. Set-up is not the same
2. **Epistemic uncertainties** of different nature

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Estimation Theory

Aim: optimal state accounting for epistemic uncertainties in experiments / CFD

Kalman Filter

Physical process at time step n described by:

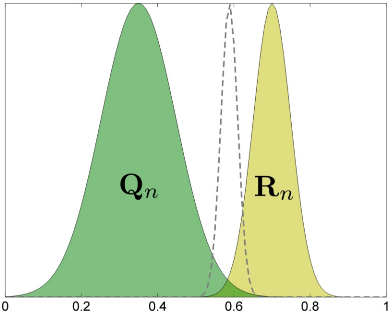
- **model**: $\mathbf{x}_n = \mathbf{H}_n \mathbf{x}_{n-1} + \mathbf{B}_n \mathbf{u}_n + \mathbf{w}_n$
- **observation**: $\mathbf{y}_n = \mathbf{H}'_n \mathbf{x}_n + \mathbf{v}_n$
- **uncertainties**: $\mathbf{w}_n = \mathcal{N}(0, \mathbf{Q_n})$, $\mathbf{v}_n = \mathcal{N}(0, \mathbf{R_n})$

The Optimal state is obtained by a **prediction step**:

- $\overline{\mathbf{x}}_{n|n-1} = \mathbf{H}_n \overline{\mathbf{x}}_{n-1|n-1} + \mathbf{B}_n \mathbf{u}_n$
- $\mathbf{P}_{n|n-1} = \mathbf{H}_n \mathbf{P}_{n-1|n-1} \mathbf{H}_n{}^T + \mathbf{Q}_n$

followed by an **update step**:

- $\mathbf{S}_n = \mathbf{H}'_n \mathbf{P}_{n|n-1} \mathbf{H}'_n{}^T + \mathbf{R}_n$
- $\mathbf{K}_n = \mathbf{P}_{n|n-1} \mathbf{H}'_n{}^T \mathbf{S}_n{}^{-1}$
- $\overline{\mathbf{x}}_{n|n} = \overline{\mathbf{x}}_{n|n-1} + \mathbf{K}_n (\mathbf{y}_n - \mathbf{H}'_n \overline{\mathbf{x}}_{n|n-1})$
- $\mathbf{P}_{n|n} = (I - \mathbf{K}_n \mathbf{H}'_n) \mathbf{P}_{n|n-1}$



Implementation in OpenFOAM

The estimator is implemented within the Pressure Implicit with Splitting of Operator (PISO) algorithm:

Momentum prediction for $\mathbf{U} = [u,\ v,\ w]$

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot (\mathbf{U} \times \mathbf{U}) - \nabla \cdot \tau = -\nabla \tilde{p} \qquad \rightarrow \qquad a_p\, \mathbf{U}_p = \mathbf{H}(\mathbf{U}) - \nabla \tilde{p}$$

$$\mathbf{P}_{n|n-1} = \mathbf{H}\, \mathbf{P}_{n-1|n-1}\, \mathbf{H}^T + \mathbf{Q}_n$$

$$\mathbf{S}_n = \mathbf{H}'_n\, \mathbf{P}_{n|n-1}\, {\mathbf{H}'_n}^T + \mathbf{R}_n, \qquad \mathbf{K}_n = \mathbf{P}_{n|n-1}\, {\mathbf{H}'_n}^T\, {\mathbf{S}_n}^{-1}$$

Correction

$$\mathbf{U} = \mathbf{U} + \mathbf{K}_n\, (\mathbf{U}_E - \mathbf{H}'_n\, \mathbf{U})$$

$$\nabla \cdot (\nabla \tilde{p}) = -\nabla \cdot (\nabla \cdot (\mathbf{U} \times \mathbf{U})) \qquad \rightarrow \qquad \nabla \tilde{p}$$

$$\mathbf{U} = \frac{\mathbf{H}(\mathbf{U})}{a_p} - \frac{\nabla \tilde{p}}{a_p}$$

$\mathbf{P}_{n|n} = (I - \mathbf{K}_n\, \mathbf{H}'_n)\, \mathbf{P}_{n|n-1}$ **(end time step)**

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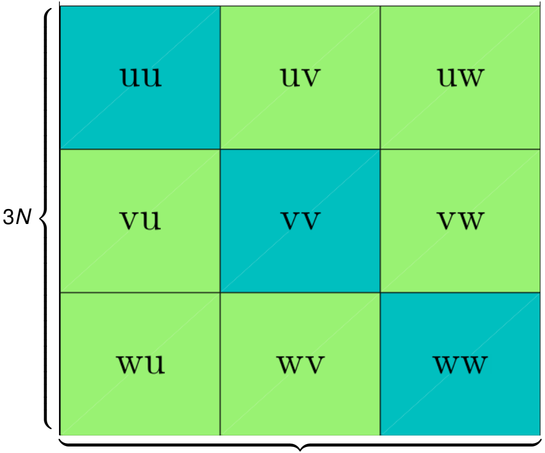
Matrices **P**, **Q**

Definition: $\mathbf{P} = cov(\mathbf{U}_i, \mathbf{U}_j)$, $\mathbf{Q} = \mathbf{G}\,\mathbf{G}^{\mathbf{T}}$, $\mathbf{G} = \mathbf{G}(\delta_x, \nu_t, \cdots)$



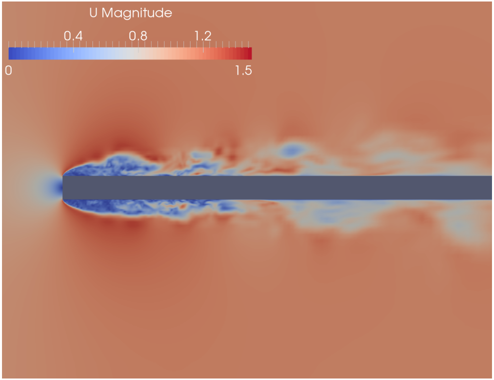
$size(\mathbf{P}) = 3N \times 3N$, full matrix.

Prohibitive resource usage for industrial applications.

Alternatives:

- Diagonal only (observator)
- Cross correlations $= 0$.
- Analytic reconstruction (Karman-Howarth equation?)

Test case: turbulent thick plate

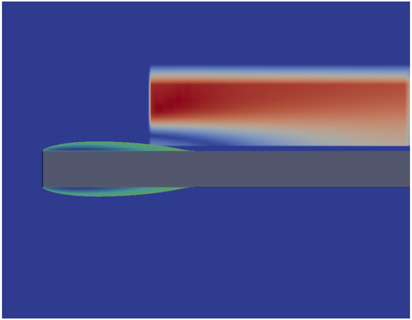


- $Re = 80000$
- DDES simulation (7×10^6 mesh elements)
- Physical domain investigated: $[-10D, 25D] \times [-8.5D, 8.5D] \times [-2.5D, 2.5D]$
- Averages over 50 recirculation times
- $2nd$ order precision in space / time
- **PIV data on plane** $[3D, 10D] \times [0.65D, 3D]$

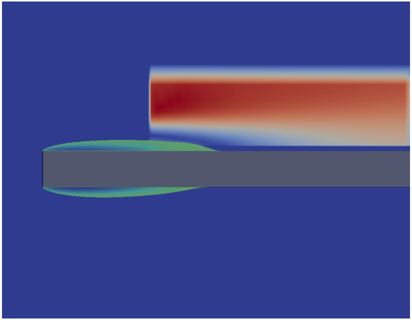
Kalman-observer for RANS

$k - \omega$ SST 2D simulations.
Physical domain $[-10D, 25D] \times [-8.5D\, 8.5D]$, 3.3×10^4 mesh elements.

Without observation:



With observation:



Significant improvement in the prediction!

Conclusions

Kalman Filter

- A Kalman filter approach has been consistently integrated in the PISO algorithm.
- Seminal results validate the accuracy and robustness of the approach.
- More development towards efficient representation of **P** and **Q**

Thank you for your attention

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