Dynamic-Multi-Scale Modeling and Simulation of Immersed Granular Flows over Obstacles

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• Classical Modeling approaches of Immersed granular Flows

• A new Dynamic-Multi-Scale Modeling approach
  (Last 7 years of personal R&D work accumulation in OpenFOAM)

• Some Results: Numerical Simulations in OpenFOAM vs Experiments

• Conclusion
Classical Modeling approaches of Immersed granular Flows

Motivation ?
Classical Modeling approaches of Immersed granular Flows

Motivation

Some applications

Nature
A mud flow
A blood flow

Industry

A concrete flow
Classical Modeling approaches of Immersed granular Flows

Motivation

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Different Scales ! Modeling ?
Classical Modeling approaches of Immersed granular Flows

Assumption

Immersed Granular Media as:
Non-Brownian Suspensions of rigid particles
(*Isothermal, incompressible, laminar*)

Fluid
(density = \( \rho_f \))
(viscosity = \( \eta_f \))

Rigid Particles
(effective diameter \( \geq 1 \) \( \mu \text{m} \))
(density = \( \rho_p \))
Classical Modeling approaches of Immersed granular Flows

Motivation (Physical phenomena)

Isodense Suspension of rigid spheres flow in a Channel
Classical Modeling approaches of Immersed granular Flows

Motivation (Physical phenomena)

*Isodense* Suspension of rigid spheres flow in a Channel

Migration of particles from higher to lower shear rate zones (Towards the centerline)
Classical Modeling approaches of Immersed granular Flows

Scale

Macroscopic

Mesoscopic

Microscopic
Advantages:

- **Scale**
  - Macroscopic
  - Mesoscopic
  - Microscopic

Real Physics is enriched
Classical Modeling approaches of Immersed granular Flows

Disadvantages

Scale
- Macroscopic
- Mesoscopic
- Microscopic

But Computation time increases
A new Dynamic-Multi-Scale Modeling approach
(Last 7 years of research work accumulation)

Dynamic-Multi-Scale Approach

Macroscopic

Mesoscopic

Microscopic
A new Dynamic-Multi-Scale Modeling approach
(Last 7 years of research work accumulation)

Dynamic-Multi-Scale Approach

Macroscopic

Mesoscopic

Microscopic

A user's choice
A new Dynamic-Multi-Scale Modeling approach


An immersed body B

Suspension of initial concentration $\phi_{\text{bulk}}$
A new Dynamic-Multi-Scale Modeling approach


Suspension flows Continuum Macroscopic Modeling is coupled to

An Immersed Boundary Method
Suspension Balance Model (SBM)
Nott and Brady (1994)
Morris and Boulay (1999)

Non-Brownian Suspensions of hard spheres

Newtonian Liquid
( density = \( \rho_f \) )
( viscosity = \( \eta_0 \) = cst. )

Monodispersed Spheres
( diameter = 2a \( \geq 1 \, \mu\text{m} \) )
( density = \( \rho_p \) )

Suspension Concentration \([\phi]\)
\[ \phi = \frac{V_s}{V_{total}} \]

Suspension Viscosity \([\eta(\phi)]\)
\[ \eta(\varphi) = \eta_0 \eta_s(\varphi) \]
\[ \eta_s(\varphi) = \left(1 - \frac{\varphi}{\varphi_m}\right)^{-2} \quad \text{(Maron & Pierce 1956)} \]
\[ 0 \leq \phi < \phi_m \quad 0.58 \leq \varphi_m \leq 0.68 \quad \text{(for spheres)} \]
Suspension Balance Model (SBM)
Nott and Brady (1994)
Morris and Boulay (1999)

Migration phenomenon is due to a flux $J \sim \nabla \cdot \Sigma^p$
Suspension flows Continuum Macroscopic Modeling

The Suspension Balance Model (SBM)

\[ \Sigma = \Sigma^f + \Sigma^p \]

**Conservation Laws**

Continuity eqn. : \[ \nabla \cdot U = 0 \quad (1) \]

Momentum eqn. : \[ \nabla \cdot \Sigma + \Delta \rho^i g \phi = 0 \quad (2) \]

Transport eqn. : \[ \frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = - \nabla \cdot J_{total} \quad (3) \]

**The Flow**

- Incompressible and viscous
- \( Re << 1 \) (non-inertial)
- \( Pe >> 1 \) (non-Brownian)
Suspension flows Cotinuum Macroscopic Modeling

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OpenFOAM®

SbmFoam Solver
Suspension flows Cotinuum Macroscopic Modeling

Viscous ReSuspension and 2D Mixing

Experiment NMR imaging
Rao et al. (2002)

SBM solver
OpenFOAM®

2D Mesh = 10000 cells | CFL << 1 | CPU time (single core Machine 1.8 GHz) ~ 5 h

A new Dynamic-Multi-Scale Modeling approach


Suspension flows Continuum Macroscopic Modeling

is coupled to

An Immersed Boundary Method
An Immersed Boundary Method coupled to the SBM


\[
\rho \frac{\partial U}{\partial t} + [\rho (U \cdot \nabla) U - \nabla \cdot \Sigma - \Delta \rho^i g \varphi] = f
\]

\[
\nabla \cdot U = 0
\]

\[
(1 - \zeta) \frac{\partial \varphi}{\partial t} + (1 - \zeta)[(U \cdot \nabla) \varphi] = -(1 - \zeta) \nabla \cdot J_t + \zeta \frac{\varphi - \varphi_B}{\Delta t}
\]
An Immersed Boundary Method coupled to the SBM


\[
\rho \frac{\partial U}{\partial t} + [\rho (U \cdot \nabla) U - \nabla \cdot \Sigma - \Delta \rho \rho^i g \varphi] = f \\
\nabla \cdot U = 0 \\
(1-\zeta) \frac{\partial \varphi}{\partial t} + (1-\zeta)[(U \cdot \nabla) \varphi] = -(1-\zeta) \nabla \cdot J_t + \zeta \frac{(\varphi - \varphi_B)}{\Delta t} \\

f = \zeta \left[ \rho \frac{\partial U}{\partial t} + \rho (U \cdot \nabla) U - \nabla \cdot \Sigma - \Delta \rho^i g \varphi + \rho \frac{(U_B - U)}{\Delta t} \right] \\

\begin{cases} 
\zeta = 1-\epsilon & \forall X(x, y, z) \in \Omega_B \\
\zeta = \epsilon & \forall X(x, y, z) \in \Omega_S 
\end{cases} 
(\epsilon \ll 1)
\]

\[
\rho = \rho_p + \rho_f \quad \text{with} \quad \rho_p = \varphi \rho_p^i \quad \text{and} \quad \rho_f = (1-\varphi) \rho_f^i 
\]
An Immersed Boundary Method coupled to the SBM


Suspension Dynamics

Effect on Immersed Body & Body/Body contacts

Rigid Body Dynamics in a Suspension Flow

Mobility of a immersed rigid bodies $B$ in a suspension flow

Suspension/Structure Interaction Force

\[ F_{SSI} = \int_{\Omega_B} \left( \nabla \cdot \Sigma + \Delta \rho^i g \varphi \right) d\Omega = -\int_{\Omega_B} \rho \frac{(1-\epsilon)}{\epsilon} \frac{(U_B - U)}{\Delta t} d\Omega \]
An Immersed Boundary Method coupled to the SBM


Suspension Dynamics
Effect on Immersed Body & Body/Body contacts

Rigid Body Dynamics laws

\[ F_T = m_B \frac{\partial U_B}{\partial t} ; \quad T_T = I_B \frac{\partial \omega_B}{\partial t} \]

\[ F_T = F_{SSI} + F_C + m_B g ; \quad T_T = T_{SSI} + T_C \]

\[ T_{SSI} = r \times F_{SSI} \]

local position vector relative to the immersed body centroid
An Immersed Boundary Method coupled to the SBM


Suspension Dynamics

Effect on Immersed Body & Body/Body contacts

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\[ F_T = m_B \frac{\partial U_B}{\partial t}; \quad T_T = I_B \frac{\partial \omega_B}{\partial t} \]

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NSCD contact laws

Validations in OpenFOAM®


Isodense suspension flow over a stationary cylinder in a wide channel

Isodense suspension flow over a stationary cylinder in a wide channel

Isodense suspension flow over a stationary cylinder in a wide channel

Validations in OpenFOAM®

Example at \( \text{Re} = \frac{\rho |\bar{U}| D}{\eta \phi_{\text{bulk}}} = 20 \)

\[
C_D = \frac{F_D}{\frac{1}{2} \rho (\bar{U} - U_B)^2 D}; \quad C_L = \frac{F_L}{\frac{1}{2} \rho (\bar{U} - U_B)^2 D}
\]

\[
F_D = \left( -\left| F_{SSI} \right|, 0, 0 \right); \quad F_L = \left( 0, -\left| F_{SSI} \right|, 0 \right)
\]

\[
\Delta p = p \left( Cx - \frac{D}{2}, Cy \right) - p \left( Cx + \frac{D}{2}, Cy \right)
\]

\[
\frac{D}{\Delta x} = 20
\]

\[
\begin{align*}
\text{Re} &= 20 \\
\alpha &= 250 \text{ \mu m} \\
\Phi_a &= 0.0001\%
\end{align*}
\]

Validations in OpenFOAM®

Example at $\text{Re} = \frac{\rho |\bar{U}| D}{\eta (\phi_{\text{bulk}})} = 20$

$C_D = \frac{F_D}{\frac{1}{2} \rho (\bar{U} - U_B)^2 D}$; $C_L = \frac{F_L}{\frac{1}{2} \rho (\bar{U} - U_B)^2 D}$

$\Delta p = p \left( C_x - \frac{D}{2}, C_y \right) - p \left( C_x + \frac{D}{2}, C_y \right)$

$F_D = \left( -F_{SSI} \right)_x, 0, 0$; $F_L = \left( 0, -F_{SSI} \right)_y, 0$

OpenFOAM® results are in good agreement with the benchmark of

S. Turek and M. Schäfer, Benchmark Computations of Laminar Flow around a Cylinder.

at $\phi_{\text{bulk}} = 10^{-4}$; $a = 250 \mu m$

OpenFOAM® results for several $\phi_{bulk}$ values

- Drag Coefficient
  - $Re=20$
  - $a=250 \mu m$
  - Equation: $\text{Drag Coefficient} = -0.00063(\phi/\phi_m)^3 + 0.0282(\phi/\phi_m) + 5.198$ (R²=93%)

- Pressure difference (Pa)
  - $Re=20$
  - $a=250 \mu m$
OpenFOAM® results for several $\phi_{bulk}$ values

Isodense suspension flow over a stationary cylinder in a micro channel


$D = 200 \mu \text{m} ; \quad \rho_s = \rho_f = 1050 \text{ kg/m}^3 ; \quad \phi_{bulk} = 8.4\% ; \quad 2a = 7 \mu \text{m}$

OpenFOAM® 2D Results

$\phi_{bulk} = 10\%$

$\text{Re} = 30 ; \quad t^* = 377.1$

$\phi_{bulk} = 10\%$

$\text{Re} = 60 ; \quad t^* = 754.2$
OpenFOAM® results for several $\phi_{\text{bulk}}$ values

Isodense suspension flow over a stationary cylinder in a micro channel

<table>
<thead>
<tr>
<th>$\phi_{\text{bulk}}$</th>
<th>Re</th>
<th>t*</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>30</td>
<td>377.1</td>
</tr>
<tr>
<td>10%</td>
<td>60</td>
<td>754.2</td>
</tr>
<tr>
<td>10%</td>
<td>90</td>
<td>263.97</td>
</tr>
</tbody>
</table>
OpenFOAM® results for several $\phi_{\text{bulk}}$ values

Isodense suspension flow over a stationary cylinder in a micro channel

Effect of Maximum packing volume fraction

$\phi_{\text{bulk}} = 10\% ; \quad \phi_m = 55\%$

Re = 90 ; $t^* = 263.97$

$\phi_{\text{bulk}} = 10\% ; \quad \phi_m = 61\%$

Re = 90 ; $t^* = 263.97$

Isodense suspension flow over a stationary cylinder in a micro channel

OpenFOAM® results are in good qualitative agreement with the experimental results of:

Conclusions

New Dynamic-Multi-Scale approach is introduced for the suspension flows of rigid particles over immersed obstacles

Successful coupling of the SBM to a DF-IBM

The new approach is developed, implemented and validated in the OpenFOAM® library
Dynamic-Multi-Scale Modeling and Simulation of Immersed Granular Flows over Obstacles

Thank you for your attention

& special thanks to the OpenFOAM community